解とけいすう
2

$(x-2)(x-3)$

2 次方程式 $2x^2 - 4x + 1 = 0$ の 2 つの解を α, β とするとき, 次の式の値を求めよ。

(1) $\alpha^4 + \beta^4$

(2) $\dfrac{\beta}{\alpha^2} + \dfrac{\alpha}{\beta^2}$

(3) $\beta - \alpha$

(4) $\sqrt{\alpha} + \sqrt{\beta}$

$\alpha + \beta = 2$

$\alpha\beta = \frac{1}{2}$

$$\begin{cases} \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta \\ \quad = 4 - 1 \\ \quad = 3 \end{cases}$$

(1) $\alpha^4 + \beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2 = 3^2 - 2\cdot\left(\frac{1}{2}\right)^2 = 9 - \frac{1}{2} = \underline{\frac{17}{2}}$ //

(2) $\dfrac{\beta}{\alpha^2} + \dfrac{\alpha}{\beta^2} = \dfrac{\beta^3+\alpha^3}{\alpha^2\beta^2} = \dfrac{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)}{\alpha^2\beta^2} = \dfrac{2\cdot\left(3-\frac{1}{2}\right)}{\left(\frac{1}{2}\right)^2} = 8\cdot\frac{5}{2} = \underline{20}$ //

(3) $\beta - \alpha = \sqrt{(\beta-\alpha)^2} = \sqrt{\alpha^2+\beta^2-2\alpha\beta} = \sqrt{3-1} = \sqrt{2}$

$(\because \beta > \alpha)$

$\therefore \beta - \alpha = -\sqrt{2}$

$(\because \beta < \alpha)$

$$\begin{cases} \beta - \alpha = \sqrt{2} & (\because \beta > \alpha) \\ \beta - \alpha = -\sqrt{2} & (\because \beta < \alpha) \end{cases}$$

(4) $\sqrt{\alpha} + \sqrt{\beta} = \sqrt{\alpha + \beta + 2\sqrt{\alpha\beta}} = \sqrt{2 + 2\sqrt{\frac{1}{2}}}$

$= \underline{\sqrt{2+\sqrt{2}}}$ //