

sqr+5

$x = \frac{2}{\sqrt{6}-2}, y = \frac{2}{\sqrt{6}+2}$ のとき、次の式の値を求めよ。

$$\begin{aligned} (1) \quad x+y &= \frac{2(\sqrt{6}+2)}{(\sqrt{6}-2)(\sqrt{6}+2)} \\ (2) \quad xy &= \frac{2(\sqrt{6}+2)}{6-4} \quad x = \sqrt{6}+2 \\ (3) \quad x^2+y^2 &= \frac{2(\sqrt{6}-2)}{(\sqrt{6}+2)(\sqrt{6}-2)} \\ (4) \quad x^3+y^3 &= \frac{2(\sqrt{6}-2)}{6-4} \quad y = \sqrt{6}-2 \\ (5) \quad \frac{x}{y} + \frac{y}{x} &= \frac{2(\sqrt{6}-2)}{(\sqrt{6}+2)(\sqrt{6}-2)} \\ (6) \quad x^5+y^5 &= \frac{2(\sqrt{6}-2)}{6-4} \end{aligned}$$

$$(1) \quad x+y = \underline{2\sqrt{6}}$$

$$(2) \quad xy = (\sqrt{6}+2)(\sqrt{6}-2) = 6-4 = \underline{2}$$

$$(3) \quad x^2+y^2 = (x+y)^2 - 2xy = (2\sqrt{6})^2 - 2 \cdot 2 = 24-4 = \underline{20}$$

$$(4) \quad x^3+y^3 = (x+y)(x^2-xy+y^2) \quad (x+y)^3 - 3xy(x+y)$$

$$= 2\sqrt{6} \cdot (20-2) = 36\sqrt{6} \quad (2\sqrt{6})^3 - 3 \cdot 2 \cdot 2\sqrt{6}$$

$$48\sqrt{6} - 12\sqrt{6} = \underline{36\sqrt{6}}$$

$$(5) \quad \frac{x}{y} + \frac{y}{x} = \frac{\sqrt{6}+2}{\sqrt{6}-2} + \frac{\sqrt{6}-2}{\sqrt{6}+2}$$

$$= \frac{(\sqrt{6}+2)^2 + (\sqrt{6}-2)^2}{(\sqrt{6}-2)(\sqrt{6}+2)}$$

$$= \frac{10+4\sqrt{6}+10-4\sqrt{6}}{6-4} = \frac{20}{2} = \underline{10}$$

$$(6) \quad x^5+y^5 = (x^2+y^2)(x^3+y^3) - x^2y^2(x+y)$$

$$= 20 \cdot 36\sqrt{6} - 2^2 \cdot 2\sqrt{6} = 720\sqrt{6} - 8\sqrt{6} = \underline{712\sqrt{6}}$$