

$x = \frac{3+\sqrt{5}}{3-\sqrt{5}}, y = \frac{3-\sqrt{5}}{3+\sqrt{5}}$  のとき、次の問いに答えよ。

1.  $x, y$  を有利化せよ。

2. 次の式の値を求めよ。

(ア)  $x+y$

(イ)  $xy$

(ウ)  $x^2+y^2$

(エ)  $x^3+y^3$

(オ)  $x^4+y^4$

$$x = \frac{(3+\sqrt{5})^2}{(3-\sqrt{5})(3+\sqrt{5})} = \frac{9+5+6\sqrt{5}}{9-5} = \frac{14+6\sqrt{5}}{4} = \frac{7+3\sqrt{5}}{2}$$

$$y = \frac{(3-\sqrt{5})^2}{(3+\sqrt{5})(3-\sqrt{5})} = \frac{9+5-6\sqrt{5}}{9-5} = \frac{14-6\sqrt{5}}{4} = \frac{7-3\sqrt{5}}{2}$$

$$1 \quad x = \frac{7+3\sqrt{5}}{2}, \quad y = \frac{7-3\sqrt{5}}{2}$$

$$2 \quad (P) \quad x+y = 7$$

$$(A) \quad xy = \left(\frac{7+3\sqrt{5}}{2}\right)\left(\frac{7-3\sqrt{5}}{2}\right) = \frac{49-45}{4} = 1$$

$$(B) \quad x^2+y^2 = (x+y)^2 - 2xy = 7^2 - 2 \cdot 1 = 47$$

$$(E) \quad x^3+y^3 = (x+y)(x^2-xy+y^2) = 7 \cdot (47-1) = 322$$

$$(O) \quad x^4+y^4 = (x^2+y^2)^2 - 2x^2y^2 = (47)^2 - 2 \cdot 1^2 = 2207$$