

2/10/04 ✓

$\sin \theta$, $\cos \theta$, $\tan \theta$ のうち、1つが次の値をとるとき、他の2つの値を求めよ。

(1) $\sin \theta = \frac{5}{6}$ (ただし, $0^\circ \leq \theta \leq 90^\circ$)

(2) $\cos \theta = -\frac{2}{3}$ (ただし, $0^\circ \leq \theta \leq 180^\circ$)

(3) $\tan \theta = \sqrt{2}$ (ただし, $0^\circ \leq \theta \leq 180^\circ$)

(1) $\sin^2 \theta + \cos^2 \theta = 1$ より

$$\cos^2 \theta = 1 - \frac{25}{36}$$

$$= \frac{11}{36}$$

$\cos \theta > 0$ より $\cos \theta = \frac{\sqrt{11}}{6}$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{5}{6}}{\frac{\sqrt{11}}{6}} = \frac{5}{\sqrt{11}} \quad \underline{\tan \theta = \frac{5}{\sqrt{11}}}$$

(2) $\sin^2 \theta = 1 - \cos^2 \theta$

$$= 1 - \frac{4}{9} = \frac{5}{9}$$

$\sin \theta > 0$ より

$$\sin \theta = \frac{\sqrt{5}}{3}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{\sqrt{5}}{3}}{-\frac{2}{3}} = -\frac{\sqrt{5}}{2} \quad \underline{\tan \theta = -\frac{\sqrt{5}}{2}}$$

(3) $\sin^2 \theta + \cos^2 \theta = 1$ より

$$\tan^2 \theta + 1 = \frac{1}{\cos^2 \theta}$$

$$(\sqrt{2})^2 + 1 = \frac{1}{\cos^2 \theta}$$

$$\cos^2 \theta = \frac{1}{3} \quad \cos \theta = \pm \frac{1}{\sqrt{3}}$$

$0^\circ \leq \theta \leq 180^\circ$ より

$\sin \theta > 0$ より

$\tan \theta > 0$ より $\cos \theta < 0$

$$\therefore \cos \theta = -\frac{1}{\sqrt{3}}$$

よって

$$\frac{\sin \theta}{\cos \theta} = \sqrt{2} \text{ より } \sin \theta = \sqrt{2} \cdot \frac{1}{\sqrt{3}}$$

$$\therefore \sin \theta = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\underline{\sin \theta = \frac{\sqrt{6}}{3}}$$