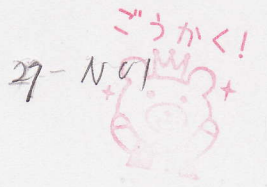




>7

-k



$f(x) = x^2 + kx$ (k は定数), $g(x) = x$ とするとき

(1) $S = \int_0^1 |f(x) - g(x)| dx$ を求めよ。

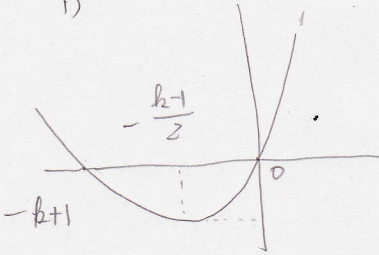
(2) $S = 1$ をみたす k の値を求めよ。

$f(x) - g(x) = x^2 + kx - x = x^2 + (k-1)x = (x + \frac{k-1}{2})^2 - \frac{(k-1)^2}{4}$ [青山学院大]

とて場合わけと行う

$f(x) - g(x) = x \{x + (k-1)\}$

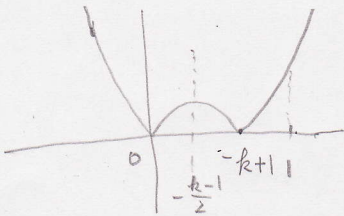
i)



$-\frac{k-1}{2} \leq 0$ である $k \geq 1$ のとき

$S = \int_0^1 x^2 + (k-1)x dx = [\frac{1}{3}x^3 + \frac{1}{2}(k-1)x^2]_0^1 = -\frac{1}{6} + \frac{1}{2}k$

ii)



左図のとき $0 \leq -k+1 \leq 1$ である

$0 \leq k \leq 1$ のとき

$S = \int_0^{-k+1} -x^2 - (k-1)x dx + \int_{-k+1}^1 x^2 + (k-1)x dx$

$= [-\frac{1}{3}x^3 - \frac{1}{2}(k-1)x^2]_0^{-k+1} + [\frac{1}{3}x^3 + \frac{1}{2}(k-1)x^2]_{-k+1}^1$

$-k+1 = d$ とし計算すると

$= -\frac{1}{3}d^3 + \frac{1}{2}d^3 + \left\{ \frac{1}{3} + \frac{1}{2}(k-1) - \left(\frac{1}{3}d^3 + \frac{1}{2}d^3 \right) \right\}$

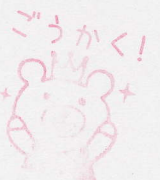
$= \frac{1}{6}d^3 + \frac{1}{2}k - \frac{1}{6} + \frac{1}{6}d^3$

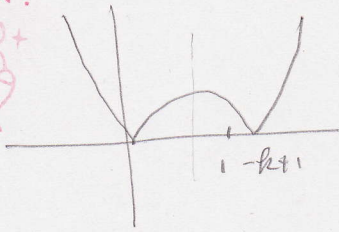
$= \frac{1}{3}d^3 + \frac{1}{2}k - \frac{1}{6}$

$= \frac{1}{3}(-k+1)^3 + \frac{1}{2}k - \frac{1}{6}$

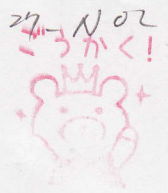
$= \frac{1}{3}(-k^3 + 3k^2 - 3k + 1) + \frac{1}{2}k - \frac{1}{6}$

$= -\frac{1}{3}k^3 + k^2 - \frac{1}{2}k + \frac{1}{6}$





29 $-k+1 \geq 1$ a b 区間 $k \leq 0$ a b 区間



$$S = \int_0^1 -x^2 - (k+1)x dx$$

$$= \left[-\frac{1}{3}x^3 - \frac{1}{2}(k+1)x^2 \right]_0^1$$

$$= \frac{1}{6} - \frac{1}{2}k$$

i) ii) iii) 区間

$$S = \begin{cases} -\frac{1}{6} + \frac{1}{2}k & (k \geq 1) \\ -\frac{1}{3}k^3 + k^2 - \frac{1}{2}k + \frac{1}{6} & (0 \leq k \leq 1) \\ \frac{1}{6} - \frac{1}{2}k & (k \leq 0) \end{cases}$$

(2)

(i) ii) $-\frac{1}{6} + \frac{1}{2}k = 1 \rightarrow -1 + 3k = 6 \quad 3k = 7 \quad k = \frac{7}{3}$ 区間外

(ii) iii) $-\frac{1}{3}k^3 + k^2 - \frac{1}{2}k + \frac{1}{6} = 1 \rightarrow h(k) = -\frac{1}{3}k^3 + k^2 - \frac{1}{2}k - \frac{5}{6}$ 区間内

$h'(k) = -k^2 + 2k - \frac{1}{2}$ $k = \frac{2 \pm \sqrt{2}}{2}$ $0 \leq k \leq 1$ の範囲で

$k = \frac{2 - \sqrt{2}}{2}$ が極小値を取る $\frac{2 - \sqrt{2}}{6} > 0$ 区間内

$0 \leq k \leq 1$ の範囲で x 軸との交点の存在しない k の値はない

(ii) iii) 区間

$\frac{1}{6} - \frac{1}{2}k = 1 \rightarrow 1 - 3k = 6 \quad -3k = 5 \quad k = -\frac{5}{3}$ 区間外

区間外

$k = -\frac{5}{3}, \frac{7}{3}$

区間外

