

次の和を求めなさい。

(1) $\sum_{k=1}^n 7^{k-1}$

(2) $\sum_{k=1}^{24} k$

(3) $\sum_{k=1}^n (2k+3)$

(4) $1 \cdot 1 + 2 \cdot 3 + 3 \cdot 5 + \dots + n(2n-1)$

(1)
$$\frac{7^n - 1}{7 - 1} = \frac{7^n - 1}{6}$$

(2)
$$\sum_{k=1}^n k = \frac{1}{2} n(n+1)$$

 $n = 24 \text{ と } 25$

$$\frac{1}{2} \cdot 24 \cdot 25 = 300$$

(3)
$$\begin{aligned} \sum_{k=1}^n (2k+3) &= 2 \sum_{k=1}^n k + 3n \\ &= 2 \cdot \frac{1}{2} n(n+1) + 3n \\ &= n^2 + n + 3n \\ &= n^2 + 4n \end{aligned}$$

(4)
$$\sum_{k=1}^n k(2k-1) = \sum_{k=1}^n (2k^2 - k)$$

$$= 2 \cdot \frac{1}{6} n(n+1)(2n+1) - \frac{1}{2} n(n+1)$$

$$= \frac{1}{3} (2n^3 + 3n^2 + n) - \frac{1}{2} (n^2 + n)$$

$$= \frac{2}{3} n^3 + \frac{1}{2} n^2 - \frac{1}{6} n$$

⇨ $n = 1, 2, 3, \dots$ と代入して確かめる

$$\frac{1}{6} n(n+1) \{ 2(2n+1) - 3 \}$$

$$\frac{1}{6} n(n+1)(4n-1)$$