

関数 $f(x) = \frac{\sqrt{2} - \cos x}{\sin x}$ ($0 < x < \pi$) の最小値を求めよ。

[東京電気大]

$$f'(x) = \frac{\sin^2 x - \cos x (\sqrt{2} - \cos x)}{\sin^2 x}$$

$$f'(x) = \frac{1 - \sqrt{2} \cos x}{\sin^2 x} \quad (0 < x < \pi) \text{ かつ}$$

$$f'(x) = 0 \text{ ならば } \cos x = \frac{1}{\sqrt{2}} \quad x = \frac{\pi}{4}$$

増減表をかきと

x	0	...	$\frac{\pi}{4}$...	π
$f'(x)$	/	-	0	+	-
$f(x)$	/	↘	極小値	↗	

$$f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2} - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 2 - 1 = 1$$

$\therefore f(x)$ は $x = \frac{\pi}{4}$ において極小値(最小値)をとる