

定積分 $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1 - \cos x) \sin^2 x dx$ を求めよ。

[弘前大]

$$\text{与式} = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sin^2 x - \sin^2 x \cos x) dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(\frac{1 - \cos 2x}{2} - \sin^2 x (\sin x)' \right) dx$$

$$= \left[\frac{1}{2} x - \frac{\sin 2x}{4} - \frac{1}{3} \sin^3 x \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \frac{1}{6} \pi - \frac{1}{4} \sin \frac{2}{3} \pi - \frac{1}{3} \sin^3 \frac{\pi}{3} - \left(\frac{1}{8} \pi - \frac{1}{4} \sin \frac{\pi}{2} - \frac{1}{3} \sin^3 \frac{1}{4} \pi \right)$$

$$= \frac{1}{6} \pi - \frac{\sqrt{3}}{8} - \frac{\sqrt{3}}{8} - \frac{1}{8} \pi + \frac{1}{4} + \frac{1}{6\sqrt{2}}$$

$$= \frac{1}{24} \pi - \frac{\sqrt{3}}{4} + \frac{\sqrt{2}}{12} + \frac{1}{4}$$