

30種分々

部分分

次の不定積分を求めよ。

$$(1) \int \frac{x^3 + x^2 + 1}{x^2(x^2 + 1)} dx \sim \frac{Ax+B}{x^2} + \frac{Cx+D}{x^2+1}$$

$$(2) \int \frac{2x^2 + 2x - 1}{x(x^3 - 1)} dx \sim \frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+x+1}$$

$$(3) \int \frac{x+2}{(x+1)^2} dx \sim \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

[基本問題]

$$\begin{aligned} (1) \quad & (Ax+B)(x^2+1) = Ax^3 + Ax + Bx^2 + B \\ & +) x^2(Cx+D) = Cx^3 + Dx^2 \\ \hline & (A+C)x^3 + (B+D)x^2 + Ax + B \quad A=D, B=1, D=0, C=1. \end{aligned}$$

$$\text{5式} = \int \left(\frac{1}{x^2} + \frac{x}{x^2+1} \right) dx = -\frac{1}{x} + \log \sqrt{x^2+1} + C$$

$$\begin{aligned} (2) \quad & A(x^3-1) + Bx(x^2+x+1) + (x^2-x)(Cx+D) \\ & Ax^3 - A + Bx^3 + Bx^2 + Bx + Cx^3 + Dx^2 - Cx^2 - Dx = (A+B+C)x^3 + (B+D-C)x^2 + (B-D)x - A \\ & \underline{A=1} \quad \underline{B+C=-1} \quad \underline{B+D-C=2} \quad \underline{B-D=2} \quad \begin{array}{l} 2B-C=4 \\ +) B+C=-1 \\ \hline 3B=3 \end{array} \quad \underline{B=1} \quad \underline{C=-2} \quad \underline{D=-1} \end{aligned}$$

$$\begin{aligned} \text{5式} &= \int \left(\frac{1}{x} + \frac{1}{x-1} - \frac{x+1}{x^2+x+1} \right) dx \\ &= \log x + \log(x-1) - \log(x^2+x+1) + C \\ &= \log \frac{|x(x-1)|}{x^2+x+1} + C \end{aligned}$$

(3)

$$\begin{aligned} \text{5式} &= \int \left\{ \frac{1}{x+1} + \frac{1}{(x+1)^2} \right\} dx \\ &= \log(x+1) - \frac{1}{x+1} + C \end{aligned}$$