

次の不定積分を求めよ。

(1)  $\int \frac{1}{\cos x} dx$

(2)  $\int \cos^3 x dx$

(3)  $\int \frac{1}{\sin x} dx$

(4)  $\int \frac{\cos x - 1}{\cos x} dx$

(5)  $\int \tan^2 x dx$

〔基本問題〕

$$(1) \text{ 与式} = \int \frac{\cos x}{\cos^3 x} dx = \int \frac{\cos x}{1 - \sin^2 x} dx \quad \sin x = t \text{ とおく} \quad \cos x dx = dt$$

$$\int \frac{\cos x}{1 - \sin^2 x} dx = \int \frac{1}{1 - t^2} dt = \int \frac{-1}{t^2 - 1} dt = \frac{1}{2} \int \left( \frac{1}{t+1} - \frac{1}{t-1} \right) dt$$

$$= \frac{1}{2} (\log|t+1| - \log|t-1|) + C$$

$$\therefore \log \sqrt{\left| \frac{\sin x + 1}{\sin x - 1} \right|} + C$$

$$(2) \text{ 与式} = \int (1 - \sin^2 x) \cos x dx \quad \sin x = t \text{ とおく} \quad \cos x dx = dt$$

$$\text{与式} = \int (1 - t^2) dt = t - \frac{1}{3} t^3 + C$$

$$\therefore \sin x - \frac{1}{3} \sin^3 x + C$$

$$(3) \text{ 与式} = \int \frac{\sin x}{\sin^3 x} dx = \int \frac{\sin x}{1 - \cos^2 x} dx \quad \cos x = t \text{ とおく} \quad -\sin x dx = dt$$

$$\text{与式} = -\int \frac{1}{1 - t^2} dt = \int \frac{1}{t^2 - 1} dt = \frac{1}{2} \int \left( \frac{1}{t-1} - \frac{1}{t+1} \right) dt$$

$$= \frac{1}{2} \log|t-1| - \frac{1}{2} \log|t+1| + C$$

$$\therefore \log \sqrt{\left| \frac{\cos x - 1}{\cos x + 1} \right|} + C$$

3C 採分 49 2/2

(4)

$$\text{与式} = \int \left(1 - \frac{1}{\cos x}\right) dx \quad \text{u) 区間制限なし}$$

$$= \underline{x - \log \left| \frac{\sin x + 1}{\sin x - 1} \right| + C}$$

(5)

$$1 + \tan^2 x = \frac{1}{\cos^2 x} \quad \text{より} \quad \tan^2 x = \frac{1}{\cos^2 x} - 1 \quad \text{区間なし}$$

$$\text{与式} = \int \left( \frac{1}{\cos^2 x} - 1 \right) dx = \underline{\tan x - x + C}$$