

30積分59

次の不定積分を求めよ。

(1) $\int_0^1 (e^{-x} + 1) dx$

(2) $\int_0^1 \frac{e^{3x}}{e^x + 1} dx$

(3) $\int_1^2 \frac{1}{e^x - 1} dx$

(4) $\int_1^3 x\sqrt{x^2 - 1} dx$

(5) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin x} dx$

(6) $\int_0^{\frac{\pi}{2}} (1 - \sin^3 x) dx$

[基本問題]

1) 互式 = $[-e^{-x} + x]_0^1 = -e^{-1} + 1 - (-1 + 0) = \underline{2 - \frac{1}{e}}$

2) $e^x = t$ とおくと $e^x dx = dt$ かつ $x: 0 \rightarrow 1$ $t: 1 \rightarrow e$

互式 = $\int_1^e \frac{t^2}{t+1} dt = \int_1^e (t-1 + \frac{1}{t+1}) dt = [\frac{1}{2}t^2 - t + \log|t+1|]_1^e$
 $= \frac{1}{2}e^2 - e + \log(e+1) - \frac{1}{2} + 1 - \log 2$
 $= \underline{\frac{1}{2}e^2 - e + \frac{1}{2} + \log \frac{e+1}{2}}$

3) $e^x = t$ とおくと $e^x dx = dt$ $x: 1 \rightarrow 2$ $t: e \rightarrow e^2$ かつ

互式 = $\int_e^{e^2} \frac{e^x}{e^x(e^x-1)} dx = \int_e^{e^2} \frac{1}{t(t-1)} dt = \int_e^{e^2} (\frac{1}{t-1} - \frac{1}{t}) dt$
 $= [\log(t-1) - \log t]_e^{e^2} = \log(e^2-1) - \log e^2 - \log(e-1) + \log e$
 $= \log \frac{(e+1)(e-1)}{e^2} = \underline{\log \frac{e+1}{e}}$

4) $x^2 - 1 = t$ $2x dx = dt$ $x: 1 \rightarrow 3$ $t: 0 \rightarrow 8$

互式 = $\frac{1}{2} \int_1^3 2x\sqrt{x^2-1} dx = \frac{1}{2} \int_0^8 \sqrt{t} dt = \frac{1}{2} \cdot \frac{2}{3} [t^{\frac{3}{2}}]_0^8$
 $= \frac{1}{3} \cdot 16\sqrt{2} = \underline{\frac{16}{3}\sqrt{2}}$

5) $1 + \sin x = t$ とおくと $\cos x dx = dt$ かつ $x: 0 \rightarrow \frac{\pi}{2}$ $t: 1 \rightarrow 2$

互式 = $\int_1^2 \frac{1}{t} dt = [\log t]_1^2 = \underline{\log 2}$

6) 互式 = $\int_0^{\frac{\pi}{2}} (1 - \sin x)(1 - \cos^2 x) dx$

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$= \int_0^{\frac{\pi}{2}} (1 - \sin x + \sin x \cos^2 x) dx = [x + \cos x - \frac{1}{3} \cos^3 x]_0^{\frac{\pi}{2}}$
 $= \frac{\pi}{2} - (1 - \frac{1}{3}) = \underline{\frac{\pi}{2} - \frac{2}{3}}$