3C 積分60

訂正

0

次の不定積分を求めよ。

(1) $\displaystyle\int_0^1 \frac{x^2+1}{x+1}\,dx$ (2) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{1+\sin x}\,dx$

(3) $\displaystyle\int_1^{\frac{\pi}{6}} \sin^2 x\cos^3 x\,dx$

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(1)

$$与式 = \int_0^1 \left(x-1+\frac{2}{x+1}\right)dx = \left[\frac{x^2}{2}-x+2\log|x+1|\right]_0^1$$

$$= \frac{1}{2}-1+2\log 2 = \underline{2\log 2-\frac{1}{2}}$$

(2)

$$与式 = \int_0^{\frac{\pi}{4}} \frac{1-\sin x}{(1+\sin x)(1-\sin x)} = \int_0^{\frac{\pi}{4}} \frac{1-\sin x}{\cos^2 x}\,dx$$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{1}{\cos^2 x}-\frac{\sin x}{\cos^2 x}\right)dx = \left[\tan x-\frac{1}{\cos x}\right]_0^{\frac{\pi}{4}}$$

$$= 1-\sqrt{2}+1 = \underline{2-\sqrt{2}}$$

(3) $\displaystyle\int_0^{\frac{\pi}{6}} \sin^2 x(1-\sin^2 x)\cos x\,dx \cdots ①$ $\sin x = t$ とおくと $\cos x\,dx = dt$

$x: 0 \to \frac{\pi}{6}$ $t: 0 \to \frac{1}{2}$

$$\therefore ① = \int_0^{\frac{1}{2}} t^2(1-t^2)\,dt = \left[\frac{t^3}{3}-\frac{t^5}{5}\right]_0^{\frac{1}{2}} = \frac{1}{24}-\frac{1}{160}$$

$$= \frac{20}{480}-\frac{3}{480} = \underline{\frac{17}{480}}$$