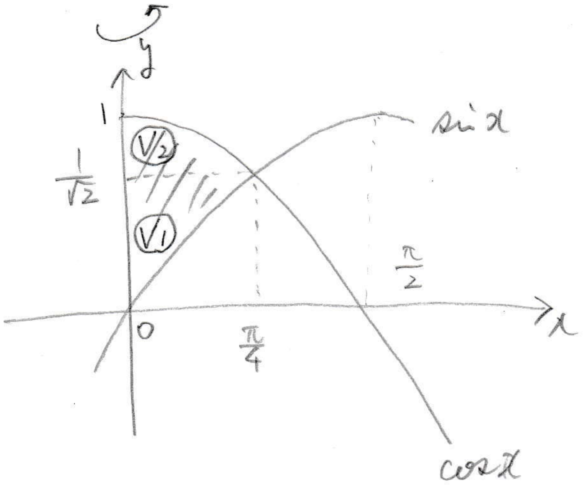
$0 \leqq x \leqq \frac{\pi}{4}$ のとき, 2つの曲線 $y=\sin x$, $y=\cos x$ および y 軸で囲まれる部分を, y 軸の周りに回転してできる回転体の体積を求めよ。 〔宮崎大〕



求める体積を V_1, V_2 として V_1 : $0 \leqq y \leqq \frac{1}{\sqrt{2}}$, V_2 : $\frac{1}{\sqrt{2}} \leqq y \leqq 1$ で分けて考えると

$$V_1 = \pi\int_0^{\frac{1}{\sqrt{2}}} x^2\,dy = \pi\int_0^{\frac{\pi}{4}} x^2\frac{dy}{dx}\,dx = \pi\int_0^{\frac{\pi}{4}} x^2\cos x\,dx$$

$$V_2 = \pi\int_{\frac{1}{\sqrt{2}}}^{1} x^2\,dy = \pi\int_{\frac{\pi}{4}}^{0} x^2\frac{dy}{dx}\,dx = \pi\int_{\frac{\pi}{4}}^{0} x^2\cdot(-\sin x)\,dx = \pi\int_0^{\frac{\pi}{4}} x^2\sin x\,dx$$

求める体積 V は $V=V_1+V_2$ より

$$V = \pi\int_0^{\frac{\pi}{4}} x^2(\overset{-\cos x+\sin x}{\sin x+\cos x})\,dx = \pi\left[x^2(\sin x-\cos x)\right]_0^{\frac{\pi}{4}} - 2\pi\int_0^{\frac{\pi}{4}} x(\sin x-\cos x)\,dx$$

$$= 2\pi\int_0^{\frac{\pi}{4}} x(\overset{\sin x+\cos x}{\cos x-\sin x})\,dx = 2\pi\left[x(\sin x+\cos x)\right]_0^{\frac{\pi}{4}} - 2\pi\int_0^{\frac{\pi}{4}}(\sin x+\cos x)\,dx$$

$$= 2\pi\cdot\frac{\sqrt{2}}{4}\pi - 2\pi\left[-\cos x+\sin x\right]_0^{\frac{\pi}{4}} = \frac{\sqrt{2}}{2}\pi - 2\pi\{0-(-1)\}$$

$$= \frac{\sqrt{2}}{2}\pi^2 - 2\pi$$

$$\underline{\frac{\sqrt{2}}{2}\pi^2 - 2\pi}$$