

fukuso |v| -10-1

次の方程式を解け。偏角 $\theta$ の範囲は  $0 \leq \theta < 2\pi$  とする

$$z^2 = 1 + \sqrt{3}i$$

$$|z|^2 = 2 \quad \text{or} \quad |z| = \sqrt{2}$$

$$z = \sqrt{2} (\cos \theta + i \sin \theta) \quad \text{と仮定}$$

$$z^2 = 2 (\cos \theta + i \sin \theta)^2$$

$$= 2 (\cos 2\theta + i \sin 2\theta)$$

$$= 2 \left( \frac{1}{2} + \frac{\sqrt{3}}{2} i \right)$$

$$= 2 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$2\theta = \frac{\pi}{3} + 2k\pi \quad \text{or} \quad \theta = \frac{\pi}{6} + k\pi \quad (k \text{ は整数})$$

$$0 \leq \theta < 2\pi \quad \text{では}$$

$$\theta = \frac{\pi}{6}, \frac{7}{6}\pi$$

よって

$$z = \sqrt{2} \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) = \sqrt{2} \left( \frac{\sqrt{3}}{2} + \frac{1}{2} i \right)$$

$$z = \sqrt{2} \left( \cos \frac{7}{6}\pi + i \sin \frac{7}{6}\pi \right) = \sqrt{2} \left( -\frac{\sqrt{3}}{2} - \frac{1}{2} i \right)$$

したがって

$$z = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} i, -\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} i$$