



次の極限を求めよ。

$$\lim_{n \rightarrow \infty} \left\{ \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \cdots \left(1 - \frac{1}{n}\right) \right\}^2 (1 + 2 + \cdots + n)$$

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$$\lim_{n \rightarrow \infty} \left( \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdots \frac{n-1}{n} \right)^2 (1 + 2 + \cdots + n)$$

$$\frac{1}{2} \cdot \frac{\cancel{2}}{\cancel{2}} \cdot \frac{\cancel{3}}{\cancel{3}} \cdot \frac{\cancel{4}}{\cancel{4}} \cdots \frac{\cancel{n-2}}{\cancel{n-2}} \cdot \frac{\cancel{n-1}}{n} = \frac{1}{n} \text{ となり!}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n^2} \cdot \frac{1}{2} n(n+1)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{2} \left(1 + \frac{1}{n}\right)$$

$$= \frac{1}{2}$$