

次の極限值を求めよ。

(1)  $\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+4}-2}$

[近畿大]

$$\frac{x}{\sqrt{x+4}-2} = \frac{x(\sqrt{x+4}+2)}{x+4-4} = \sqrt{x+4}+2$$

$$\therefore \lim_{x \rightarrow 0} \sqrt{x+4}+2 = \underline{4}$$

(2)  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2}-1}{x^2}$

[工学院大]

$$\frac{\sqrt{1+x^2}-1}{x^2} = \frac{1+x^2-1}{x^2(\sqrt{1+x^2}+1)} = \frac{1}{\sqrt{1+x^2}+1}$$

$$\therefore \lim_{x \rightarrow 0} \frac{1}{\sqrt{1+x^2}+1} = \underline{\frac{1}{2}}$$

極限 2/2

$$(3) \lim_{x \rightarrow 1} \frac{x^3 - 2x^2 - x + 2}{x^2 - 1}$$

$$\begin{array}{r} x^2 - x - 2 \quad (x-2)(x+1) \\ x-1 \overline{) x^3 - 2x^2 - x + 2} \\ \underline{x^3 - x^2} \phantom{- x + 2} \\ \phantom{x^3} -x^2 - x + 2 \\ \phantom{x^3} \phantom{-x^2} -x + 2 \\ \phantom{x^3} \phantom{-x^2} \phantom{-x} 2 \end{array}$$

[豊橋技大]

$$\frac{x^3 - 2x^2 - x + 2}{x^2 - 1} = \frac{(x+1)(x-1)(x-2)}{(x+1)(x-1)} = (x-2)$$

$$\lim_{x \rightarrow 1} (x-2) = -1$$

$$(4) \lim_{x \rightarrow 0} \frac{\sqrt{x^2 - x + 1} - 1}{\sqrt{1+x} - \sqrt{1-x}}$$

[東京電機大]

$$\begin{aligned} \frac{\sqrt{x^2 - x + 1} - 1}{\sqrt{1+x} - \sqrt{1-x}} &= \frac{x(x-1)}{(\sqrt{1+x} - \sqrt{1-x})(\sqrt{x^2 - x + 1} + 1)} \\ &= \frac{x(x-1)(\sqrt{1+x} + \sqrt{1-x})}{2x(\sqrt{x^2 - x + 1} + 1)} \\ &= \frac{(x-1)(\sqrt{1+x} + \sqrt{1-x})}{2(\sqrt{x^2 - x + 1} + 1)} \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{(x-1)(\sqrt{1+x} + \sqrt{1-x})}{2(\sqrt{x^2 - x + 1} + 1)} = \frac{-1 \cdot 2}{2 \cdot 2} = -\frac{1}{2}$$