

$$\lim_{n \rightarrow \infty} |a_n| = 3$$

次の無限級数の収束, 発散を調べ, 収束するときはその和を求めよ。

$$\frac{3}{1 \cdot 2} + \frac{3}{2 \cdot 3} + \frac{3}{3 \cdot 4} + \cdots + \frac{3}{n(n+1)} + \cdots$$

$$\begin{aligned} S_n &= 3 \left( 1 - \frac{1}{2} \right) + 3 \left( \frac{1}{2} - \frac{1}{3} \right) + 3 \left( \frac{1}{3} - \frac{1}{4} \right) \\ &\quad + \cdots + 3 \left( \frac{1}{n} - \frac{1}{n+1} \right) \\ &= 3 \left( 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{n} - \frac{1}{n+1} \right) \\ &= 3 \left( 1 - \frac{1}{n+1} \right) \end{aligned}$$

$$\therefore \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} 3 \left( 1 - \frac{1}{n+1} \right) = 3$$