

無限級数 27

$S_n = \frac{1}{1^2+2} + \frac{1}{2^2+4} + \dots + \frac{1}{n^2+2n}$ のとき, $\lim_{n \rightarrow \infty} S_n = \square$ である。 [近畿大]

$$\begin{aligned} \frac{1}{n^2+2n} &= \frac{1}{n(n+2)} \\ &= \frac{1}{2} \left(\frac{1}{n} - \frac{1}{n+2} \right) \text{ とおくと} \end{aligned}$$

$$\begin{aligned} S_n &= \frac{1}{2} \left(1 - \frac{1}{3} \right) + \frac{1}{2} \left(\frac{1}{2} - \frac{1}{4} \right) + \frac{1}{2} \left(\frac{1}{3} - \frac{1}{5} \right) + \frac{1}{2} \left(\frac{1}{4} - \frac{1}{6} \right) + \dots \\ &\quad \dots + \frac{1}{2} \left(\frac{1}{n-2} - \frac{1}{n} \right) + \frac{1}{2} \left(\frac{1}{n-1} - \frac{1}{n+1} \right) + \frac{1}{2} \left(\frac{1}{n} - \frac{1}{n+2} \right) \\ &= \frac{1}{2} + \frac{1}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)} \end{aligned}$$

$$\begin{aligned} \therefore \lim_{n \rightarrow \infty} S_n &= \lim_{n \rightarrow \infty} \left\{ \frac{1}{2} + \frac{1}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)} \right\} \\ &= \frac{3}{4} \end{aligned}$$