無限級数 27

$S_n = \dfrac{1}{1^2+2} + \dfrac{1}{2^2+4} + \cdots + \dfrac{1}{n^2+2n}$ のとき，$\lim\limits_{n\to\infty} S_n = \boxed{\phantom{00}}$ である。　〔近畿大〕

$$\frac{1}{n^2+2n} = \frac{1}{n(n+2)}$$

$$= \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right)$$ となるので

$$S_n = \frac{1}{2}\left(1 - \cancel{\frac{1}{3}}\right) + \frac{1}{2}\left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \frac{1}{2}\left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \frac{1}{2}\left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \cdots$$

$$\cdots + \frac{1}{2}\left(\cancel{\frac{1}{n-2}} - \cancel{\frac{1}{n}}\right) + \frac{1}{2}\left(\cancel{\frac{1}{n-1}} - \frac{1}{n+1}\right) + \frac{1}{2}\left(\cancel{\frac{1}{n}} - \frac{1}{n+2}\right)$$

$$= \frac{1}{2} + \frac{1}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)}$$

$$\therefore \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left\{\frac{1}{2} + \frac{1}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)}\right\}$$

$$= \frac{3}{4}$$