

次の無限級数の和を求めよ。

(1)  $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots$

(2)  $\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \dots$

(3)  $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \dots$

[練習問題]

1)

$$a_n = \frac{1}{(2n-1)(2n+1)} = \frac{1}{2} \left( \frac{1}{2n-1} - \frac{1}{2n+1} \right)$$

$$S_n = \sum_{k=1}^n a_k = \frac{1}{2} \left\{ \left( 1 - \frac{1}{3} \right) + \left( \frac{1}{3} - \frac{1}{5} \right) + \left( \frac{1}{5} - \frac{1}{7} \right) + \dots + \left( \frac{1}{2n-1} - \frac{1}{2n+1} \right) \right\}$$

$$= \frac{1}{2} \left( 1 - \frac{1}{2n+1} \right)$$

$$\lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \frac{1}{2} \left( 1 - \frac{1}{2n+1} \right) = \frac{1}{2}$$

2)

$$a_n = \frac{1}{n(n+1)(n+2)}$$

$$S_n = \sum_{k=1}^n a_k = \frac{1}{2} \left\{ \frac{1}{k(k+1)} - \frac{1}{(k+1)(k+2)} \right\}$$

$$= \frac{1}{2} \left\{ \left( \frac{1}{2} - \frac{1}{6} \right) + \left( \frac{1}{6} - \frac{1}{12} \right) + \left( \frac{1}{12} - \frac{1}{20} \right) + \dots + \left( \frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \right) \right\}$$

$$= \frac{1}{2} \left\{ \frac{1}{2} - \frac{1}{(n+1)(n+2)} \right\}$$

$$\lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \frac{1}{2} \left\{ \frac{1}{2} - \frac{1}{(n+1)(n+2)} \right\} = \frac{1}{4}$$

3)

$$a_n = \frac{1}{n(n+1)} = \frac{2}{n(n+1)}$$

$$S_n = \sum_{k=1}^n \frac{2}{n(n+1)} = 2 \sum_{k=1}^n \left( \frac{1}{n} - \frac{1}{n+1} \right)$$

$$= 2 \left\{ \left( 1 - \frac{1}{2} \right) + \left( \frac{1}{2} - \frac{1}{3} \right) + \left( \frac{1}{3} - \frac{1}{4} \right) + \dots + \left( \frac{1}{n} - \frac{1}{n+1} \right) \right\}$$

$$= 2 \left( 1 - \frac{1}{n+1} \right)$$

$$\lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} 2 \left( 1 - \frac{1}{n+1} \right) = 2$$