2次方程式 $x^2 - 2x + 5 = 0$ の2つの解を α, β とするとき、次の式の値を求めよ。

(1) $\alpha^3 + \beta^3$

(2) $\dfrac{\beta}{\alpha+1} + \dfrac{\alpha}{\beta+1}$

(3) $(\alpha - \beta)^2$

$\alpha + \beta = 2$

$\alpha\beta = 5$

$(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

でも可

(1) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$ …①

ここで $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 4 - 10 = -6$

∴ ①は $\alpha^3 + \beta^3 = 2 \cdot (-6-5) = -22$

(2) $\dfrac{\beta(\beta+1) + \alpha(\alpha+1)}{(\alpha+1)(\beta+1)} = \dfrac{\alpha^2 + \beta^2 + \alpha + \beta}{\alpha\beta + \alpha + \beta + 1} = \dfrac{-6+2}{5+2+1} = -\dfrac{4}{8} = -\dfrac{1}{2}$

(3) $(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$

$= \alpha^2 + \beta^2 - 2\alpha\beta$

$= -6 - 10$

$= -16$