sqrt 11.

(1) $\dfrac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$ の分母を有利化せよ。

(2) $\dfrac{1}{1+\sqrt{2}+\sqrt{3}}+\dfrac{1}{1+\sqrt{2}-\sqrt{3}}-\dfrac{1}{1-\sqrt{2}+\sqrt{3}}-\dfrac{1}{1-\sqrt{2}-\sqrt{3}}$ を簡単にしなさい。

2つずつペアにして通分すると効率的に計算できる

(1) $$\frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{(\sqrt{2}+\sqrt{3}+\sqrt{5})(\sqrt{2}+\sqrt{3}-\sqrt{5})}$$

$$= \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{(\sqrt{2}+\sqrt{3})^2-5}$$

$$= \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{5+2\sqrt{6}-5}$$

$$= \frac{\sqrt{6}(\sqrt{2}+\sqrt{3}-\sqrt{5})}{2\sqrt{6}\cdot\sqrt{6}}$$

$$= \frac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{12}$$

(2) 与式 $= \dfrac{(1+\sqrt{2}-\sqrt{3})+(1+\sqrt{2}+\sqrt{3})}{(1+\sqrt{2}+\sqrt{3})(1+\sqrt{2}-\sqrt{3})} - \left\{ \dfrac{(1-\sqrt{2}-\sqrt{3})+(1-\sqrt{2}+\sqrt{3})}{(1-\sqrt{2}+\sqrt{3})(1-\sqrt{2}-\sqrt{3})} \right\}$

$$= \frac{2+2\sqrt{2}}{(1+\sqrt{2})^2-3} - \frac{2-2\sqrt{2}}{(1-\sqrt{2})^2-3}$$

$$= \frac{2+2\sqrt{2}}{3+2\sqrt{2}-3} - \frac{2-2\sqrt{2}}{3-2\sqrt{2}-3}$$

$$= \frac{2+2\sqrt{2}}{2\sqrt{2}} + \frac{2-2\sqrt{2}}{2\sqrt{2}}$$

$$= \frac{1+\sqrt{2}}{\sqrt{2}} + \frac{1-\sqrt{2}}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}}$$

$$= \sqrt{2}$$

$\dfrac{\sqrt{2}+2}{2} + \dfrac{\sqrt{2}-2}{2} = \sqrt{2}$ としてもOK