

$x = \frac{2}{\sqrt{5}+1}$ ,  $y = \frac{\sqrt{5}+1}{2}$  のとき、次の式の値を求めよ。

$$(1) \quad x^2y + xy^2 \quad x = \frac{2(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)} = \frac{2(\sqrt{5}-1)}{4} = \frac{\sqrt{5}-1}{2}$$

$$(2) \quad x^2 + y^2$$

$$(3) \quad x^3 + y^3$$

$$(4) \quad x^5 + y^5$$

$$x+y = \frac{\sqrt{5}-1}{2} + \frac{\sqrt{5}+1}{2} = \sqrt{5}$$

$$xy = \left(\frac{\sqrt{5}-1}{2}\right)\left(\frac{\sqrt{5}+1}{2}\right) = \frac{5-1}{4} = 1$$

$$(1) \quad xy(x+y) = 1 \cdot \sqrt{5} = \underline{\underline{\sqrt{5}}}$$

$$(2) \quad x^2+y^2 = (x+y)^2 - 2xy \\ = (\sqrt{5})^2 - 2 \cdot 1 = 5 - 2 = \underline{\underline{3}}$$

$$(3) \quad \begin{aligned} x^3+y^3 &= (x+y)^3 - 3xy(x+y) & x^3+y^3 \\ &= (\sqrt{5})^3 - 3 \cdot 1 \cdot \sqrt{5} & = (x+y)(x^2-xy+y^2) \\ &= 5\sqrt{5} - 3\sqrt{5} = \underline{\underline{2\sqrt{5}}} & = \sqrt{5} \cdot (3-1) \\ & & = \underline{\underline{2\sqrt{5}}} \end{aligned}$$

$$(4) \quad \begin{aligned} x^5+y^5 &= (x^2+y^2)(x^3+y^3) - x^2y^2(x+y) \\ &= 3 \cdot 2\sqrt{5} - 1^2 \cdot \sqrt{5} \\ &= 6\sqrt{5} - \sqrt{5} \\ &= \underline{\underline{5\sqrt{5}}} \end{aligned}$$