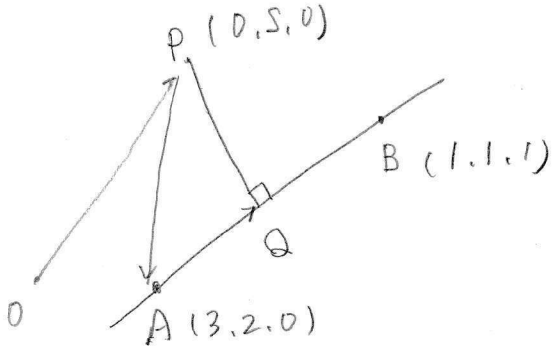

(1)

$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ}$

$\overrightarrow{PA} = (3, 2-s, 0)$

$\overrightarrow{AQ} = t\overrightarrow{AB}$

$= t(-2,-1,1)$

$\overrightarrow{PQ} = \begin{pmatrix} 3 \\ 2-s \\ 0 \end{pmatrix} + \begin{pmatrix} -2t \\ -t \\ t \end{pmatrix}$

$= (3-2t,\ 2-s-t,\ t)$

$\overrightarrow{PQ} \perp \overrightarrow{AB}$ より

$-2(3-2t) - (2-s-t) + t = 0$

$-6+4t-2+s+t+t=0$

$6t + s - 8 = 0$

$6t = 8-s$

$t = \frac{8-s}{6}$

$\overrightarrow{PQ} = \left(3 - \frac{8-s}{3},\ 2-s-\frac{8-s}{6},\ \frac{8-s}{6}\right)$

$= \left(\frac{1+s}{3},\ \frac{4-5s}{6},\ \frac{8-s}{6}\right)$

$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

$= \left(\frac{1+s}{3},\ \frac{4-5s}{6}+s,\ \frac{8-s}{6}\right)$

$\therefore Q\left(\frac{1+s}{3},\ \frac{4+s}{6},\ \frac{8-s}{6}\right)$

(2)

$f(s) = \sqrt{\left(\frac{1+s}{3}\right)^2 + \left(s-\frac{4+s}{6}\right)^2 + \left(\frac{8-s}{6}\right)^2}$

$= \sqrt{\frac{1}{9}(1+2s+s^2) + \frac{1}{36}(16-40s+25s^2) + \frac{1}{36}(64-16s+s^2)}$

$= \frac{1}{6}\sqrt{4+8s+4s^2+16-40s+25s^2+64-16s+s^2}$

$= \frac{1}{6}\sqrt{30s^2-48s+84}$

$= \frac{\sqrt{6}}{6}\sqrt{5s^2-8s+14}$

$= \frac{\sqrt{6}}{6}\sqrt{5\left(s-\frac{4}{5}\right)^2 + \frac{54}{5}}$

よって $s = \frac{4}{5}$ のとき最小となる