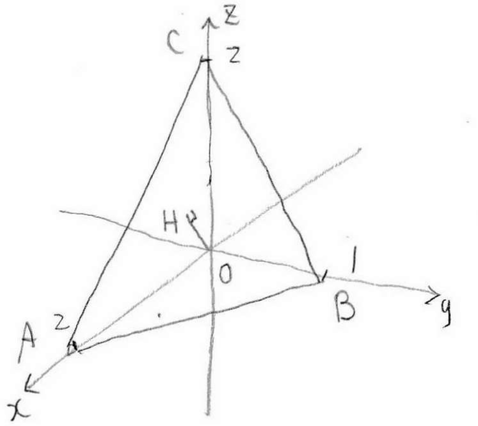
(1) A(2, 0, 0), B(0, 1, 0), C(0, 0, 2)



$$\overrightarrow{OH} = s\overrightarrow{OA} + t\overrightarrow{OB} + u\overrightarrow{OC} \quad (\because s+t+u=1)$$

$$= s\begin{pmatrix}2\\0\\0\end{pmatrix} + t\begin{pmatrix}0\\1\\0\end{pmatrix} + u\begin{pmatrix}0\\0\\2\end{pmatrix}$$

$$= (2s,\ t,\ 2u)$$

$\overrightarrow{AB} = (-2, 1, 0)$

$\overrightarrow{AC} = (-2, 0, 2)$

$\overrightarrow{AB} \perp \overrightarrow{OH}$ より $-4s + t = 0 \quad t = 4s$

$\overrightarrow{AC} \perp \overrightarrow{OH}$ より $-4s + 4u = 0 \quad u = s$

よって $\overrightarrow{OH} = s\overrightarrow{OA} + 4s\overrightarrow{OB} + s\overrightarrow{OC}$ $\quad s+4s+s=1$ より $s = \frac{1}{6},\ t = \frac{2}{3},\ u = \frac{1}{6}$

$\therefore \overrightarrow{OH} = \left(\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right)$ $\qquad$ $\underline{H\left(\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right)}$

(2)

$\overrightarrow{AB} = (-2, 1, 0)$ より $|\overrightarrow{AB}| = \sqrt{(-2)^2 + 1^2 + 0^2} = \sqrt{5}$

$\overrightarrow{AC} = (-2, 0, 2)$ より $|\overrightarrow{AC}| = \sqrt{(-2)^2 + 0^2 + 2^2} = 2\sqrt{2}$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 4$

よって △ABCの面積Sは $S = \frac{1}{2}\sqrt{(\sqrt{5} \cdot 2\sqrt{2})^2 - 4^2} = \frac{1}{2}\sqrt{40-16} = \sqrt{6}$

$|\overrightarrow{OH}| = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = \sqrt{\frac{1}{9} + \frac{4}{9} + \frac{1}{9}} = \frac{\sqrt{6}}{3}$

したがって求める体積は $\frac{1}{3}\sqrt{6} \cdot \frac{\sqrt{6}}{3} = \frac{2}{3}$ $\qquad$ $\underline{\frac{2}{3}}$

別解 上図より OA=2 OB1 OC=2 より

$\frac{1}{3} \cdot \frac{1}{2} \cdot 2 \cdot 1 \cdot 2 = \underline{\frac{2}{3}}$