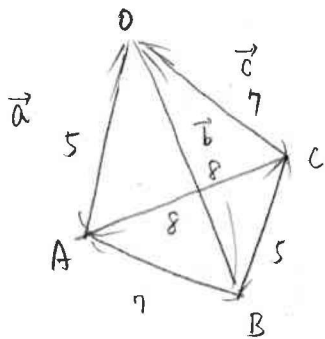


(1)



$$\angle AOB = \theta_1 \text{ とする}$$

$$49 = 25 + 64 - 2 \cdot 5 \cdot 8 \cos \theta_1$$

$$80 \cos \theta_1 = 40$$

$$\cos \theta_1 = \frac{1}{2}$$

$$\angle BOC = \theta_2 \text{ とする}$$

$$25 = 64 + 49 - 2 \cdot 8 \cdot 7 \cos \theta_2$$

$$2 \cdot 8 \cdot 7 \cos \theta_2 = 88$$

$$\cos \theta_2 = \frac{11}{14}$$

$$\angle AOC = \theta_3 \text{ とする}$$

$$64 = 25 + 49 - 2 \cdot 5 \cdot 7 \cos \theta_3$$

$$2 \cdot 5 \cdot 7 \cos \theta_3 = 10$$

$$\cos \theta_3 = \frac{1}{7}$$

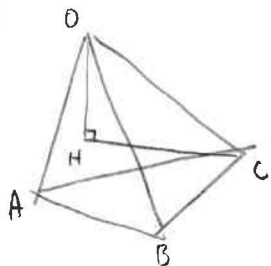
$$\therefore \vec{a} \cdot \vec{b} = 5 \cdot 8 \cdot \frac{1}{2} = 20$$

$$\vec{b} \cdot \vec{c} = 8 \cdot 7 \cdot \frac{11}{14} = 44$$

$$\vec{c} \cdot \vec{a} = 5 \cdot 7 \cdot \frac{1}{7} = 5$$

$$\vec{a} \cdot \vec{a} = 20, \vec{b} \cdot \vec{b} = 44, \vec{c} \cdot \vec{c} = 5$$

(2)



$$\vec{CH} = -\vec{OC} + s\vec{OA} + t\vec{OB} \quad \vec{OH}$$

$$= s\vec{a} + t\vec{b} - \vec{c}$$

$$\vec{OH} \perp \vec{AC} \text{ 故に } \vec{CH} \cdot \vec{a} = 0 \text{ となる}$$

$$s|\vec{a}|^2 + t\vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{a} = 0$$

$$25s + 20t = 5 \quad \text{--- ①}$$

$$\vec{CH} \perp \vec{AB} \text{ 故に } \vec{CH} \cdot \vec{b} = 0 \text{ となる}$$

$$s\vec{a} \cdot \vec{b} + t|\vec{b}|^2 - \vec{b} \cdot \vec{c} = 0$$

$$20s + 64t = 44 \quad \text{--- ②}$$

①, ②より

$$100s + 80t = 20$$

$$\text{--- } 100s + 320t = 220$$

$$-240t = -200$$

$$t = \frac{5}{6}$$

$$25s + \frac{50}{3} = 5$$

$$25s = -\frac{35}{3}$$

$$s = -\frac{7}{15}$$

$$\therefore \vec{OH} = -\frac{7}{15}\vec{a} + \frac{5}{6}\vec{b}$$

(3)

$$\vec{OH} = -\vec{c} + \vec{OH}$$

$$= -\frac{7}{15}\vec{a} + \frac{5}{6}\vec{b} - \vec{c}$$

$$|\vec{OH}|^2 = \frac{49}{225}|\vec{a}|^2 - \frac{7}{9}\vec{a} \cdot \vec{b} + \frac{25}{36}|\vec{b}|^2 - 2\vec{c} \cdot \left(-\frac{7}{15}\vec{a} + \frac{5}{6}\vec{b}\right) + |\vec{c}|^2$$

$$= \frac{49}{9} - \frac{140}{9} + \frac{400}{9} + \frac{14}{3} - \frac{220}{3} + 49$$

$$= \frac{132}{9}$$

$$\vec{OH} = \frac{2\sqrt{33}}{3}$$

(4)

$$\Delta OAB = \frac{1}{2} \sqrt{|\vec{OA}|^2 |\vec{OB}|^2 - (\vec{OA} \cdot \vec{OB})^2}$$

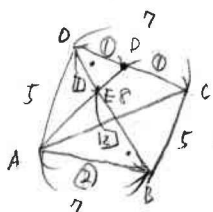
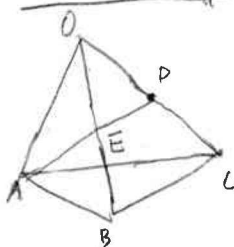
$$= \frac{1}{2} \sqrt{25 \cdot 64 - 20^2}$$

$$= \frac{1}{2} \cdot 20\sqrt{3}$$

$$= 10\sqrt{3}$$

$$\therefore V_1 = \frac{1}{3} \cdot 10\sqrt{3} \cdot \frac{2\sqrt{33}}{3} \text{ とあるが}$$

$$\text{実際 } V_1 = \frac{20\sqrt{11}}{3}$$



$$OE : EB = 1 : 2$$

$$OD : DC = 1 : 1 \text{ (中点)}$$

$$\text{求める体積は } \frac{1 \times 1}{3 \times 2} \times V_1$$

$$\therefore V_2 = \frac{1}{6} V_1 = \frac{10\sqrt{11}}{9}$$

四面体 OABC は平行四辺形