ⅡB 三角関数 24

$0 < \theta < \dfrac{\pi}{2}$, $\sin\theta\cos\theta = \dfrac{1}{8}$ のとき, 次の値を求めよ。

(1) $\sin\theta + \cos\theta = \boxed{}$

(2) $\sin^3\theta + \cos^3\theta = \boxed{}$

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(1)

$\overset{\frac{1}{8}}{\sin\theta\cos\theta} = (\sin\theta + \cos\theta)^2 - 1 - \overset{\frac{1}{8}}{\sin x \cos x}$ なので

$(\sin\theta + \cos\theta)^2 = \dfrac{1}{8} + 1 + \dfrac{1}{8}$

$(\sin\theta + \cos\theta)^2 = \dfrac{5}{4}$

$\sin\theta + \cos\theta = \pm\dfrac{\sqrt{5}}{2}$ $\quad 0 < \theta < \dfrac{\pi}{2}$ より $\sin\theta > 0$ $\cos\theta > 0$ より

$\sin\theta + \cos\theta = \dfrac{\sqrt{5}}{2}$

(2)

$\sin^3\theta + \cos^3\theta = (\sin\theta + \cos\theta)(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)$

$= \dfrac{\sqrt{5}}{2} \cdot \left(1 - \dfrac{1}{8}\right)$

$= \dfrac{\sqrt{5}}{2} \cdot \dfrac{7}{8}$

$\therefore = \dfrac{7\sqrt{5}}{16}$