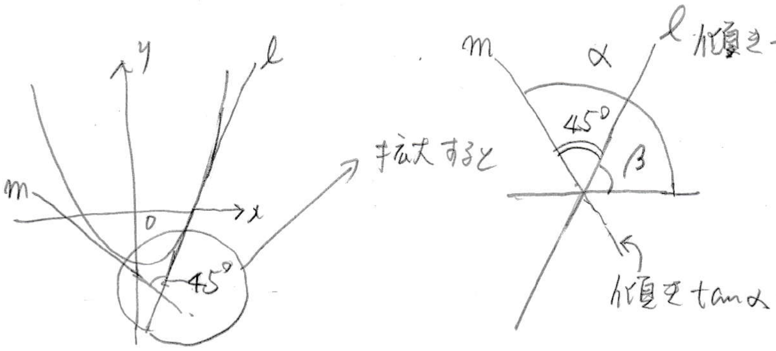
(1)

$C: y=\frac{1}{2}x^2-\frac{1}{2}$　　接点Aの座標は $A(3,4)$

$y'=x$ なので 接線lは

$l: y=3(x-3)+4 \rightarrow \underline{y=3x-5}$

(2)



加法定理より

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

$$\frac{\tan 45^\circ}{1}=\frac{\tan\alpha-3}{1+3\tan\alpha}$$

$1+3\tan\alpha=\tan\alpha-3$

$2\tan\alpha=-4$

$\tan\alpha=-2$　　よって $\underline{-2}$

(3)

$y'=x$ で　mの傾きが-2なので

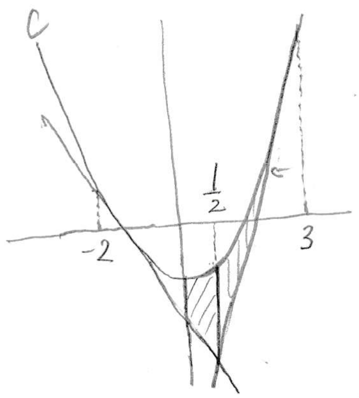
Bのx座標は-2　つまり $B\left(-2,\frac{3}{2}\right)$ であるから

mの式は

$y=-2(x+2)+\frac{3}{2} \rightarrow y=-2x-\frac{5}{2}$

mとlの式の交点を求めると　$\underline{P\left(\frac{1}{2},-\frac{7}{2}\right)}$

(4)



求める面積Sは

$$S=\int_0^{\frac{1}{2}}\left\{\left(\frac{1}{2}x^2-\frac{1}{2}\right)-\left(-2x-\frac{5}{2}\right)\right\}dx+\int_{\frac{1}{2}}^{3}\left\{\left(\frac{1}{2}x^2-\frac{1}{2}\right)-(3x-5)\right\}dx$$

$$=\int_0^{\frac{1}{2}}\left(\frac{1}{2}x^2+2x+2\right)dx+\int_{\frac{1}{2}}^{3}\left(\frac{1}{2}x^2-3x+\frac{9}{2}\right)dx$$

$$=\left[\frac{1}{6}x^3+x^2+2x\right]_0^{\frac{1}{2}}+\left[\frac{1}{6}x^3-\frac{3}{2}x^2+\frac{9}{2}x\right]_{\frac{1}{2}}^{3}$$

$$=\left(\cancel{\frac{1}{48}}+\frac{1}{4}+1\right)+\left(\frac{9}{2}-\cancel{\frac{27}{2}}+\cancel{\frac{27}{2}}\right)-\left(\cancel{\frac{1}{48}}-\frac{3}{8}+\frac{9}{4}\right)$$

$$=\frac{31}{8}$$

$\underline{\frac{31}{8}}$