$\displaystyle\sum_{k=1}^{n} \frac{1}{\sqrt{k}+\sqrt{k+1}}$ を簡単にしなさい。

$$\frac{\sqrt{k}-\sqrt{k+1}}{(\sqrt{k}+\sqrt{k+1})(\sqrt{k}-\sqrt{k+1})} = \frac{\sqrt{k}-\sqrt{k+1}}{-1} = \sqrt{k+1}-\sqrt{k}$$

よって

$$与式 = \sum_{k=1}^{n} (\sqrt{k+1}-\sqrt{k})$$

$$= (\cancel{\sqrt{2}}-1) + (\cancel{\sqrt{3}}-\cancel{\sqrt{2}}) + (\cancel{2}-\cancel{\sqrt{3}}) + \cdots + (\cancel{\sqrt{n}}-\cancel{\sqrt{n-1}}) + (\sqrt{n+1}-\cancel{\sqrt{n}})$$

$$= -1+\sqrt{n+1}$$

$$\underline{A.\ \sqrt{n+1}-1}$$

$S_n = \dfrac{1}{2!} + \dfrac{2}{3!} + \dfrac{3}{4!} + \cdots + \dfrac{n}{(n+1)!}$ を求めなさい。

$$\frac{n}{(n+1)!} = \frac{(n+1)-1}{(n+1)!} = \frac{n+1}{(n+1)!} - \frac{1}{(n+1)!} = \frac{1}{n!} - \frac{1}{(n+1)!}$$

$$S_n = \sum_{k=1}^{n} \left\{ \frac{1}{k!} - \frac{1}{(k+1)!} \right\} = \left(1 - \cancel{\frac{1}{2!}}\right) + \left(\cancel{\frac{1}{2!}} - \cancel{\frac{1}{3!}}\right)$$

$$+ \left(\cancel{\frac{1}{3!}} - \cancel{\frac{1}{4!}}\right) + \cdots + \left(\cancel{\frac{1}{n!}} - \frac{1}{(n+1)!}\right)$$

よって

$$\underline{1 - \frac{1}{(n+1)!}}$$