

$\sum_{k=1}^n \frac{2k-1}{3^k}$ を求めよ。

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$$S = \sum_{k=1}^n \frac{2k-1}{3^k} \quad \text{と置く}$$

$$S = \frac{1}{3} + \frac{2}{3^2} + \frac{5}{3^3} + \frac{7}{3^4} + \dots + \frac{2n-1}{3^n}$$

$$- \frac{1}{3} S = \frac{1}{3^2} + \frac{3}{3^3} + \frac{5}{3^4} + \dots + \frac{2n-3}{3^n} + \frac{2n-1}{3^{n+1}}$$

$$\frac{2}{3} S = \frac{1}{3} + \frac{2}{3^2} + \frac{2}{3^3} + \frac{2}{3^4} + \dots + \frac{2}{3^n} - \frac{2n-1}{3^{n+1}}$$

$$\frac{2}{3} S = -\frac{1}{3} + \frac{2}{3} + \frac{2}{3^2} + \frac{2}{3^3} + \frac{2}{3^4} + \dots + \frac{2}{3^n} - \frac{2n-1}{3^{n+1}}$$

$$\frac{2}{3} S = -\frac{1}{3} + 2 \left\{ \frac{\frac{1}{3}(1-\frac{1}{3^n})}{1-\frac{1}{3}} \right\} - \frac{2n-1}{3^{n+1}}$$

$$\frac{2}{3} S = -\frac{1}{3} + 1 - \frac{1}{3^n} - \frac{2n-1}{3^{n+1}}$$

$$2S = -1 + 3 - \frac{1}{3^n} - \frac{2n-1}{3^n}$$

$$2S = 2 + \frac{-3-2n+1}{3^n}$$

$$2S = 2 + \frac{-2n-2}{3^n}$$

$$S = 1 + \frac{-n-1}{3^n}$$

と置く