(1)

$$c_{n+1} = a_{n+1} + b_{n+1}$$

$$= (2a_n + b_n) + \frac{1}{2}(a_n + 3b_n)$$

$$= \frac{5}{2}a_n + \frac{5}{2}b_n$$

$$= \frac{5}{2}(a_n + b_n)$$

$$= \frac{5}{2}c_n$$

$$\therefore \underline{c_{n+1} = \frac{5}{2}c_n}$$

(2)

$$c_1 = a_1 + b_1 = 3$$

$$\therefore \underline{c_n = 3\left(\frac{5}{2}\right)^{n-1}}$$

(3)

$$a_n + b_n = 3\cdot\left(\frac{5}{2}\right)^{n-1}$$

$$-)\ 2a_n + b_n = a_{n+1}$$

$$-a_n = 3\cdot\left(\frac{5}{2}\right)^{n-1} - a_{n+1}$$

より

$$a_{n+1} - a_n = 3\left(\frac{5}{2}\right)^{n-1}$$

$$\therefore a_n = a_1 + \sum_{k=1}^{n-1} 3\left(\frac{5}{2}\right)^{n-1}$$

$$= 1 + \frac{3\left\{\left(\frac{5}{2}\right)^{n-1} - 1\right\}}{\frac{5}{2} - 1}$$

$$= 1 + 2\left\{\left(\frac{5}{2}\right)^{n-1} - 1\right\}$$

$$= 2\cdot\left(\frac{5}{2}\right)^{n-1} - 1$$

$$a_n + b_n = 3\left(\frac{5}{2}\right)^{n-1}$$

$$-)\ a_n + 3b_n = 2b_{n+1}$$

$$-2b_n = 3\left(\frac{5}{2}\right)^{n-1} - 2b_{n+1}$$

$$2b_{n+1} - 2b_n = 3\left(\frac{5}{2}\right)^{n-1}$$

$$b_{n+1} - b_n = \frac{3}{2}\left(\frac{5}{2}\right)^{n-1}$$

$$\therefore b_n = b_1 + \sum_{k=1}^{n-1} \frac{3}{2}\left(\frac{5}{2}\right)^{n-1}$$

$$= 2 + \frac{\frac{3}{2}\left\{\left(\frac{5}{2}\right)^{n-1} - 1\right\}}{\frac{5}{2} - 1}$$

$$= \left(\frac{5}{2}\right)^{n-1} + 1$$

以上より

$$a_n = 2\cdot\left(\frac{5}{2}\right)^{n-1} - 1$$

$$b_n = \left(\frac{5}{2}\right)^{n-1} + 1$$