3C 定積分

$$\int \frac{x}{\sqrt{x+1}}\,dx = \square$$

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$\sqrt{x+1} - \frac{1}{\sqrt{x+1}}$

$\therefore \frac{x}{\sqrt{x+1}} = \sqrt{x+1} - \frac{1}{\sqrt{x+1}}$ より

与式

$$\int \left(\sqrt{x+1} - \frac{1}{\sqrt{x+1}}\right) dx$$

$$= \int \left\{(x+1)^{\frac{1}{2}} - (x+1)^{-\frac{1}{2}}\right\} dx$$

$$= \frac{2}{3}(x+1)^{\frac{3}{2}} - 2(x+1)^{\frac{1}{2}}$$

$$= \frac{2}{3}(x+1)^{\frac{3}{2}} - 2\sqrt{x+1} + C \quad (\because C\text{は積分定数})$$

$\therefore$ $\frac{2}{3}(x+1)^{\frac{3}{2}} - 2\sqrt{x+1} + C$ ($\because C$は積分定数)