

3C 積分50

次の不定積分を求めよ。

(1) $\int \frac{1}{1 - \sin x} dx$

(2) $\int \frac{\cos^3}{1 - \sin x} dx$

(3) $\int \sin x (\sin x - \cos x) dx$

(4) $\int \sin^5 x dx$

(5) $\int \cos^5 x dx$

[基本問題]

(1) 両辺 $\int \frac{1 + \sin x}{1 - \sin^2 x} dx = \int \frac{1 + \sin x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} dx + \int \frac{\sin x}{\cos^2 x} dx$
 $= \tan x + \frac{1}{\cos x} + C$

(2) 両辺 $\int \frac{\cos^3 x (1 + \sin x)}{\cos^2 x} dx = \int \cos x (1 + \sin x) dx$ $\int (1 + \sin x) dx = x - \cos x + C$
 $1 + \sin x = t$ とおくと $\cos x dx = dt$

$\int t dt = \frac{1}{2} t^2 + C = \frac{1}{2} (1 + \sin x)^2 + C$

(3) 両辺 $\int (\sin^2 x - \sin x \cos x) dx$

$\cos^2 x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$ $\sin^2 x = \frac{1 - \cos 2x}{2}$
 $2\sin x \cos x = \sin 2x$ $\sin x \cos x = \frac{1}{2} \sin 2x$

両辺 $\int \left(\frac{1 - \cos 2x}{2} - \frac{1}{2} \sin 2x \right) dx$
 $= \frac{1}{2} x - \frac{1}{4} \sin 2x + \frac{1}{4} \cos 2x + C$

(4) 両辺 $\int \sin^4 x \cos x dx = \int (1 - \cos^2 x)^2 \cos x dx$ $\cos x = t$ とおくと $-\sin x dx = dt$
 $= -\int (1 - t^2)^2 dt = \int (1 - 2t^2 + t^4) dt = \frac{1}{5} t^5 - \frac{2}{3} t^3 + t + C$
 $= \frac{1}{5} \cos^5 x - \frac{2}{3} \cos^3 x + \cos x + C$

(5) 両辺 $\int \cos^4 x \cos x dx = \int (1 - \sin^2 x)^2 \cos x dx$ $\sin x = t$ とおくと $\cos x dx = dt$
 $= \int (1 - t^2)^2 dt = \int (1 - 2t^2 + t^4) dt = \frac{1}{5} t^5 - \frac{2}{3} t^3 + t + C$

$\frac{1}{5} \sin^5 x - \frac{2}{3} \sin^3 x + \sin x + C$