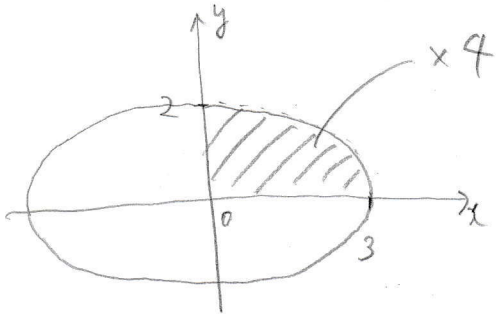
3C積分72

楕円 $x = 3\cos t, y = 2\sin t$ の面積を求めよ。　〔基本問題〕

求める面積Sとすると



$S = 4\int_0^3 y\,dx$　　$dx = -3\sin t\,dt$

$x: 0 \to 3$　$t: \frac{\pi}{2} \to 0$

$= 4\int_{\frac{\pi}{2}}^{0} 2\sin t \cdot -3\sin t\,dt$

$= 24\int_0^{\frac{\pi}{2}} \sin^2 t\,dt$

$= 24\int_0^{\frac{\pi}{2}} \frac{1-\cos 2t}{2}\,dt$

$= 12\left\{ [x]_0^{\frac{\pi}{2}} - \left[\frac{\sin 2t}{2}\right]_0^{\frac{\pi}{2}} \right\}$

$= 12 \cdot \frac{\pi}{2}$

$= 6\pi$

$\therefore$ 6π

$\sin^2 t =$

$\cos^2 t - \sin^2 t = \cos 2t$

$1 - 2\sin^2 t = \cos 2t$

$\sin^2 t = \frac{1-\cos 2t}{2}$