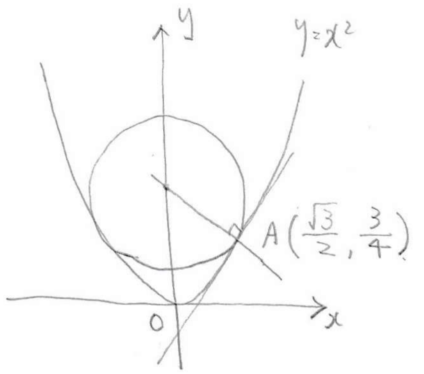
2013 愛知大

(1)



$y'=2x$

Aにおける接線の方程式は

$$y=\sqrt{3}\left(x-\frac{\sqrt{3}}{2}\right)+\frac{3}{4}$$

となるので点Aにおける法線は

$$y=-\frac{1}{\sqrt{3}}\left(x-\frac{\sqrt{3}}{2}\right)+\frac{3}{4} \rightarrow y=-\frac{1}{\sqrt{3}}x+\frac{5}{4}$$

となるので円の中心は $\left(0, \frac{5}{4}\right)$

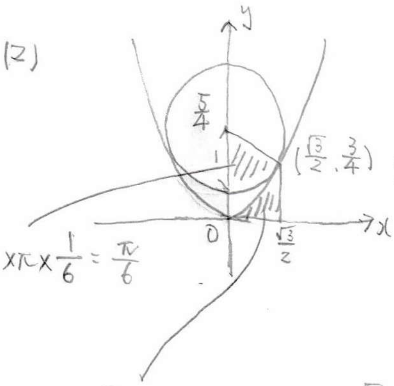
これと点Aとの距離は

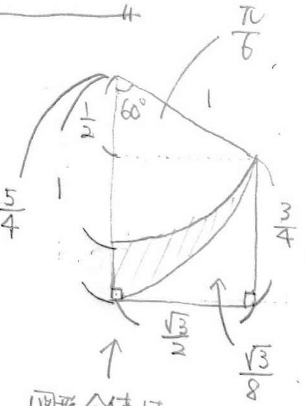
$$\sqrt{\left(\frac{\sqrt{3}}{2}-0\right)^2+\left(\frac{3}{4}-\frac{5}{4}\right)^2}=\sqrt{\frac{3}{4}+\frac{1}{4}}=1$$

円Cの方程式は

$$\underline{x^2+\left(y-\frac{5}{4}\right)^2=1}$$

(2)





図形全体は台形

$$\underbrace{\left(\frac{5}{4}+\frac{3}{4}\right)}_{2}\times\frac{\sqrt{3}}{2}\times\frac{1}{2}=\frac{\sqrt{3}}{2}$$

$$\int_0^{\frac{\sqrt{3}}{2}} x^2\,dx=\left[\frac{1}{3}x^3\right]_0^{\frac{\sqrt{3}}{2}}$$

$$=\frac{\sqrt{3}}{8}$$

よって

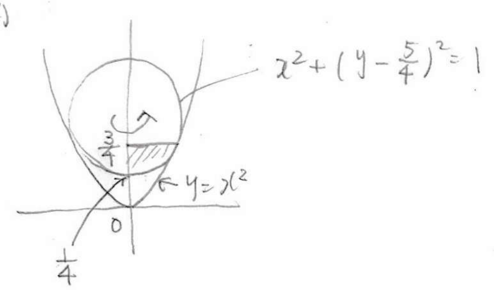
$$\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{8}-\frac{\pi}{6}=\frac{3\sqrt{3}}{8}-\frac{\pi}{6}$$

したがってこれを2倍して

$$2\left(\frac{3\sqrt{3}}{8}-\frac{\pi}{6}\right)=\frac{3\sqrt{3}}{4}-\frac{\pi}{3}$$

$$\underline{\frac{3\sqrt{3}}{4}-\frac{\pi}{3}}$$

(3)



$$\pi\int_0^{\frac{3}{4}} \underset{y=x^2}{x^2}\,dy-\pi\int_{\frac{1}{4}}^{\frac{3}{4}} \underset{x^2+\left(y-\frac{5}{4}\right)^2=1}{x^2}\,dy$$

$$=\pi\int_0^{\frac{3}{4}} y\,dy-\pi\int_{\frac{1}{4}}^{\frac{3}{4}}\left\{1-\left(y-\frac{5}{4}\right)^2\right\}dy$$

$$=\pi\left[\frac{1}{2}y^2\right]_0^{\frac{3}{4}}-\pi\left[y-\frac{1}{3}\left(y-\frac{5}{4}\right)^3\right]_{\frac{1}{4}}^{\frac{3}{4}}$$

$$=\frac{9}{32}\pi-\pi\left\{\left(\frac{3}{4}+\frac{1}{24}\right)-\left(\frac{1}{4}+\frac{1}{3}\right)\right\}$$

$$=\frac{9}{32}\pi-\frac{5}{24}\pi$$

$$=\frac{7}{96}\pi$$

$$\underline{\frac{7}{96}\pi}$$