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$x = a(2\cos\theta + \cos 2\theta + 1)$,

$y = a(2\sin\theta + \sin 2\theta)$

(1) $\dfrac{dx}{d\theta} = a(-2\sin\theta - 2\sin 2\theta)$

$= -2a\sin\theta(1+2\cos\theta)$

$\dfrac{dy}{d\theta} = a(2\cos\theta + 2\cos 2\theta)$

$= a(2\cos\theta + 4\cos^2\theta - 2)$

$= 2a(\cos\theta + 1)(2\cos\theta - 1)$

$$\left.\begin{aligned} \frac{dx}{d\theta} &= -2a\sin\theta(1+2\cos\theta) \\ \frac{dy}{d\theta} &= 2a(\cos\theta+1)(2\cos\theta-1) \end{aligned}\right\} \text{(答)}$$

(2) (1)より $\dfrac{dx}{d\theta} = 0$ とすると $\theta = 0, \frac{2}{3}\pi, \pi$ $(0 \leqq \theta \leqq \pi)$

増減表をかくと

θ	0	$\cdots$	$\frac{2}{3}\pi$	$\cdots$	π
$\frac{dx}{d\theta}$	0	$-$	0	$+$	0
x	$4a$	↘	$-\frac{1}{2}a$	↗	0

よって x は $\theta = 0$ のとき最大で $4a$

このとき $y = 0$ $\quad\therefore \theta = 0$ で $A(4a, 0)$

また $\theta = \frac{2}{3}\pi$ のとき最小で $-\frac{1}{2}a$

このとき $y = \frac{\sqrt{3}}{2}a$ $\quad\therefore \theta = \frac{2}{3}\pi$ で $C(-\frac{1}{2}a, \frac{\sqrt{3}}{2}a)$

(1)より $\dfrac{dy}{d\theta} = 0$ とすると $\theta = \frac{\pi}{3}, \pi$ $(0 \leqq \theta \leqq \pi)$

増減表をかくと

θ	0	$\cdots$	$\frac{\pi}{3}$	$\cdots$	π
$\frac{dy}{d\theta}$		$+$	0	$-$	
y	0	↗	$\frac{3\sqrt{3}}{2}a$	↘	0

よって y は $\theta = \frac{\pi}{3}$ のとき最大で $\frac{3\sqrt{3}}{2}a$

このとき $x = \frac{3}{2}a$ $\quad\therefore \theta = \frac{\pi}{3}$ で $B(\frac{3}{2}a, \frac{3\sqrt{3}}{2}a)$

以上より

$$\left.\begin{aligned} &A(4a, 0),\ \theta = 0 \\ &B\left(\tfrac{3}{2}a, \tfrac{3\sqrt{3}}{2}a\right),\ \theta = \tfrac{\pi}{3} \\ &C\left(-\tfrac{1}{2}a, \tfrac{\sqrt{3}}{2}a\right)\ \theta = \tfrac{2}{3}\pi \end{aligned}\right\} \text{(答)}$$

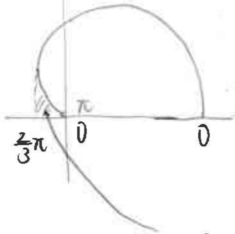
$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$-\underline{\ \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta\ }$

$-2\sin\alpha\sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$

(3)



$\displaystyle\int_{\frac{2}{3}\pi}^{0} y\,dx - \int_{\frac{2}{3}\pi}^{\pi} y\,dx$

$\displaystyle= -\int_{0}^{\frac{2}{3}\pi} y\,dx - \int_{\frac{2}{3}\pi}^{\pi} y\,dx$

$\displaystyle= -\int_{0}^{\pi} y\,dx$

$\displaystyle= -\int_{0}^{\pi} y\,\frac{dx}{d\theta}\cdot d\theta$

$\displaystyle= \int_{0}^{\pi} a(2\sin\theta + \sin 2\theta)\cdot a(2\sin\theta + 2\sin 2\theta)\,d\theta$

$\displaystyle= a^2\int_{0}^{\pi} (4\sin^2\theta + 6\sin\theta\sin 2\theta + 2\sin^2 2\theta)\,d\theta$

$\displaystyle= a^2\int_{0}^{\pi} \left\{4\cdot\frac{1-\cos 2\theta}{2} + 3(\cos\theta - \cos 3\theta) + 2\cdot\frac{1-\cos 4\theta}{2}\right\} d\theta$

$\displaystyle= a^2\int_{0}^{\pi} (-\cos 4\theta - 3\cos 3\theta - 2\cos 2\theta + 3\cos\theta + 3)\,d\theta$

$\displaystyle= a^2\left[-\frac{1}{4}\sin 4\theta - \sin 3\theta - \sin 2\theta + 3\sin\theta + 3\theta\right]_{0}^{\pi}$

$= 3\pi a^2$

よって求める面積は $3\pi a^2$