

(1) 1の3乗根を求めなさい。

$$0 \leq \theta < 2\pi$$

(2) 1の6乗根を求めなさい。

$$(1) z = r(\cos \theta + i \sin \theta) \quad r > 0, \quad 0 \leq \theta < 2\pi \text{ と } \theta \neq \pi$$

$$z^3 = 1 \quad (*)$$

$$r^3 (\cos \theta + i \sin \theta)^3 = 1$$

$$r^3 (\cos 3\theta + i \sin 3\theta) = 1 (\cos 0 + i \sin 0)$$

$$r^3 = 1 \quad r = 1$$

$$3\theta = 0 + 2\pi k$$

$$\theta = \frac{2}{3}\pi k \quad k = 0, 1, 2.$$

$$\theta = 0 \text{ かつ } z = 1$$

$$\theta = \frac{2}{3}\pi \text{ かつ } z = \cos \frac{2}{3}\pi + i \sin \frac{2}{3}\pi = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$\theta = \frac{4}{3}\pi \text{ かつ } z = \cos \frac{4}{3}\pi + i \sin \frac{4}{3}\pi = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$(2) z = r(\cos \theta + i \sin \theta) \quad r > 0, \quad 0 \leq \theta < 2\pi \text{ と } \theta \neq \pi$$

$$r^6 (\cos \theta + i \sin \theta)^6 = 1$$

$$r^6 (\cos 6\theta + i \sin 6\theta) = 1 (\cos 0 + i \sin 0)$$

$$6\theta = 0 + 2\pi k \quad r^6 = 1 \quad r = 1 \quad r > 0 \text{ かつ}$$

$$\theta = \frac{1}{3}\pi k$$

$$k = 0, 1, 2, 3, 4, 5$$

$$k = 0 \text{ かつ } z = 1$$

$$k = 1 \text{ かつ } z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$k = 2 \text{ かつ } z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$k = 3 \text{ かつ } z = -1$$

$$k = 4 \text{ かつ } z = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$k = 5 \text{ かつ}$$

$$z = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

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$$\text{以上より } 1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -1, -\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$$