

30 積分 27

次の極限值を求めよ。

$$\lim_{n \rightarrow \infty} \left\{ \frac{1}{n} + \frac{1}{n+a^2} + \frac{1}{n+2a^2} + \cdots + \frac{1}{n+(n-1)a^2} \right\}$$

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$$\text{互式} = \lim_{n \rightarrow \infty} \frac{1}{n} \left\{ 1 + \frac{1}{1+\frac{a^2}{n}} + \frac{1}{1+\frac{2}{n}a^2} + \cdots + \frac{1}{1+\frac{n-1}{n}a^2} \right\}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=0}^{n-1} \frac{1}{1+\frac{k}{n}a^2}$$

$$= \int_0^1 \frac{1}{1+a^2x} dx$$

$$= \frac{1}{a^2} [\log |1+a^2x|]_0^1$$

$$= \frac{1}{a^2} \{ \log(1+a^2) - 0 \}$$

$$\therefore \underline{\frac{1}{a^2} \log(1+a^2)}$$