次の極限値を求めよ。

$$\lim_{n \to \infty} \frac{1}{n^2}\left\{ \left(\sqrt{1}+\sqrt{n}\right)^2 + \left(\sqrt{2}+\sqrt{n}\right)^2 + \cdots + \left(\sqrt{n-1}+\sqrt{n}\right)^2 \right\}$$

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$\dot{\mp}\dot{\mathbb{X}} = \lim_{n \to \infty} \frac{1}{n}\left\{ \left(\sqrt{\frac{1}{n}}+1\right)^2 + \left(\sqrt{\frac{2}{n}}+1\right)^2 + \cdots + \left(\sqrt{\frac{n-1}{n}}+1\right)^2 \right\}$

$= \lim_{n \to \infty}\left\{ \frac{1}{n}\sum_{k=1}^{n}\left(1+\sqrt{\frac{k}{n}}\right)^2 - \underbrace{\frac{1}{n}\left(\sqrt{\frac{n}{n}}+1\right)^2}_{0 \quad n \to \infty} \right\}$

$= \int_{0}^{1}\left(1+\sqrt{x}\right)^2 dx$

$= \int_{0}^{1}\left(1+2\sqrt{x}+x\right)dx$

$= \left[x + \frac{4}{3}x^{\frac{3}{2}} + \frac{x^2}{2} \right]_{0}^{1}$

$= 1 + \frac{4}{3} + \frac{1}{2}$

$= \frac{17}{6}$