

次の極限を求めよ。

(1) $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+3} - \sqrt{n}}$
 (3) $\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n-1})$

(2) $\lim_{n \rightarrow \infty} \frac{n}{\sqrt{n+1} - 1}$
 (4) $\lim_{n \rightarrow \infty} \sqrt{n}(\sqrt{n+1} - \sqrt{n})$

[key words: 有理化]

(1) $\lim_{n \rightarrow \infty} \frac{\sqrt{n+3} + \sqrt{n}}{n+3 - n} = \lim_{n \rightarrow \infty} \frac{\sqrt{n+3} + \sqrt{n}}{3} = \infty$

(2) $\lim_{n \rightarrow \infty} \frac{n(\sqrt{n+1} + 1)}{n+1 - 1} = \lim_{n \rightarrow \infty} \sqrt{n+1} + 1 = \infty$

(3) $\lim_{n \rightarrow \infty} \frac{(n+1) - (n-1)}{\sqrt{n+1} + \sqrt{n-1}} = \lim_{n \rightarrow \infty} \frac{2}{\sqrt{n+1} + \sqrt{n-1}} = 0$

(4) $\lim_{n \rightarrow \infty} \frac{\sqrt{n}(n+1 - n)}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n+1} + \sqrt{n}}$
 $= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{n}} + 1}$
 $= \frac{1}{2}$