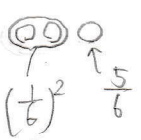


4)  $n \geq 3$  1個のサイコロを  $n$  回振る.


2013 年 1 月

$6n$  個のうち  $2$  の  $2$  回で連続で出る確率  $P_n$


$P_3$


 $2 \times \left(\frac{1}{6}\right)^2 \cdot \frac{5}{6} = \frac{5}{108} \dots P_3$

$P_4$


 $3 \times \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^2 = \frac{25}{432} \dots P_4$

(2)


 $P_n = {}^{n-1}C_1 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{n-2}$

$P_n = (n-1) \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^{n-2}$

∴ 7 と 3

$$P_{n+1} - \frac{5}{6} P_n = n \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^{n-1} - (n-1) \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^{n-1}$$

$$= \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{n-1}$$

∴ 7 と 3 は示された。

(3)

$S_n = P_3 + P_4 + \dots + P_n$  とする

$$S_n = 2 \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right) + 3 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 + 4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3 + \dots + (n-1) \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{n-2}$$

$$\rightarrow \frac{5}{6} S_n = 2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 + 3 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3 + \dots + (n-2) \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{n-2} + (n-1) \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{n-1}$$

$$\frac{1}{6} S_n = 2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right) + \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 + \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3 + \dots + \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{n-2} - (n-1) \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{n-1}$$

$$\left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right) + \sum_{k=1}^{n-2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^k \rightarrow \left(\frac{1}{6}\right)^2 \frac{5}{6} \cdot \frac{1 - \left(\frac{5}{6}\right)^{n-2}}{1 - \frac{5}{6}} = \frac{5}{36} \left\{ 1 - \left(\frac{5}{6}\right)^{n-2} \right\}$$

$\left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right) + \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)$   
 ← 1 と分ける

$$\therefore \frac{1}{6} S_n = \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right) + \frac{5}{36} \left\{ 1 - \left(\frac{5}{6}\right)^{n-2} \right\} - (n-1) \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{n-1}$$

$$S_n = \frac{5}{36} + \frac{5}{6} \left\{ 1 - \left(\frac{5}{6}\right)^{n-2} \right\} - (n-1) \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{n-1}$$

$$\lim_{n \rightarrow \infty} S_n = \frac{5}{36} + \frac{5}{6} = \frac{5}{36} + \frac{30}{36} = \frac{35}{36}$$

$$\therefore \lim_{n \rightarrow \infty} S_n = \frac{35}{36}$$