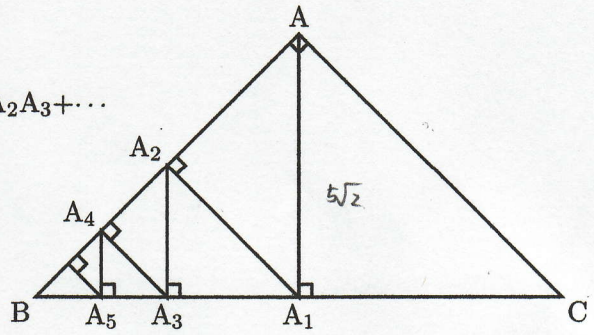
無限級数 10

直角2等辺三角形の頂点からBCに垂線AA_1を下す。A_1からABに垂線A_1A_2を下ろし，以下，右の図のように続けていくとき，$AA_1+A_1A_2+A_2A_3+\cdots$を求めよ。ただし，ACの長さは10 cmとする。



〔日本女子大〕

$AA_1 = 10\cdot\frac{1}{\sqrt{2}} = 10\left(\frac{1}{\sqrt{2}}\right)^1$

$A_1A_2 = 10\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = 10\left(\frac{1}{\sqrt{2}}\right)^2$

$A_2A_3 = 10\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = 10\left(\frac{1}{\sqrt{2}}\right)^3$

$\vdots$

$A_{n-1}A_n = 10\left(\frac{1}{\sqrt{2}}\right)^n$

$$\sum_{n=1}^{\infty} 10\left(\frac{1}{\sqrt{2}}\right)^n = 10\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{1-\frac{1}{\sqrt{2}}}$$

$$= 10\cdot\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}-1}$$

$$= 10(\sqrt{2}+1)$$

$10\sqrt{2}+10$ (cm)