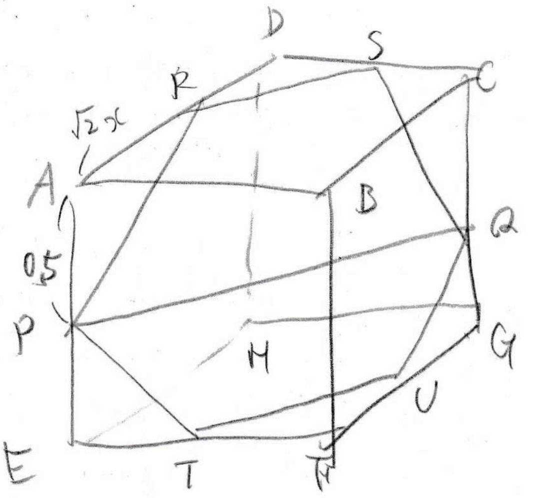
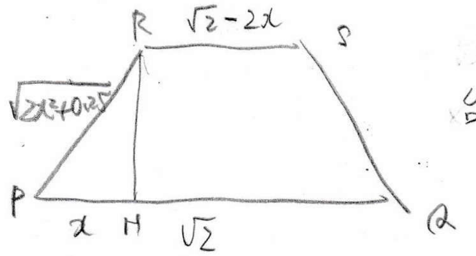
六角形RPTUQSを考える



$PR = \sqrt{2x^2+0.25}$



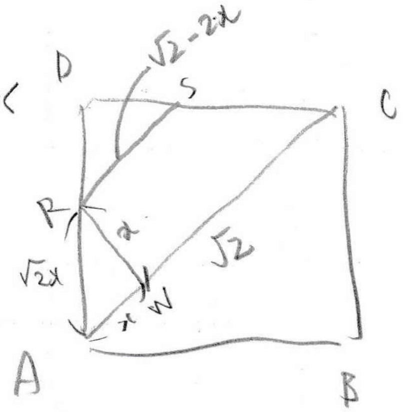
台形RPQSの2つ分が
六角形RPTUQS
の面積

$RH = \sqrt{2x^2+0.25-x^2}$

$= \sqrt{x^2+0.25}$

↓

上から見ると



RW = x とおく

$0 \leqq x \leqq \frac{\sqrt{2}}{2}$

よって

六角形の面積 $S(x)$ は

$S(x) = (\sqrt{2}-2x+\sqrt{2}) \times \sqrt{x^2+0.25} \times \frac{1}{2} \times 2$

$= (2\sqrt{2}-2x)\sqrt{x^2+0.25}$

$= 2\sqrt{(\sqrt{2}-x)^2(x^2+0.25)}$ $\quad 0 \leqq x \leqq \frac{\sqrt{2}}{2}$

ここで $f(x) = (\sqrt{2}-x)^2(x^2+0.25)$ とおくと $\quad 0 \leqq x \leqq \frac{\sqrt{2}}{2}$

$f'(x) = 2(\sqrt{2}-x)\cdot(-1)(x^2+0.25) + (\sqrt{2}-x)^2 \cdot 2x$

$= (\sqrt{2}-x)(-2x^2-0.5+2\sqrt{2}x-2x^2)$

$= (\sqrt{2}-x)(-4x^2+2\sqrt{2}x-\frac{1}{2})$

$= -\frac{1}{2}(\sqrt{2}-x)(2\sqrt{2}x-1)^2$ $\quad 0 \leqq x \leqq \frac{\sqrt{2}}{2}$ で $f'(x) < 0$

$x = \sqrt{2}, \frac{1}{2\sqrt{2}}$ $\quad$ 単調減少

x	0	…	$\frac{1}{2\sqrt{2}}$	…	$\frac{\sqrt{2}}{2}$
$f'(x)$	/	−	0	−	/
$f(x)$	0.5	↘	/	↘	$\frac{3}{8}$
$S(x)$	$\sqrt{2}$	↘	/	↘	$\frac{\sqrt{6}}{2}$

長方形 $\qquad$ ひし形

$f(x)$ は $0 \leqq x \leqq \frac{\sqrt{2}}{2}$ で単調減少なので $x=0$ で最大

$x = \frac{\sqrt{2}}{2}$ で最小となる

最大値は $x=0$ で $\sqrt{2}$ (長方形)

最小値は $x = \frac{\sqrt{2}}{2}$ で $\frac{\sqrt{6}}{2}$ (ひし形)